

Gauge Dependence in the AdS/CFT Correspondence*

H. O. Girotti^a and Victor O. Rivelles^b

^a*Instituto de Física, Universidade Federal do Rio Grande do Sul*

Caixa Postal 15051, 91501-970 - Porto Alegre, RS, Brazil

E-mail: hgirotti@if.ufrgs.br

^b*Instituto de Física, Universidade de São Paulo*

Caixa Postal 66318, 05315-970 - São Paulo, SP, Brazil

E-mail: rivelles@fma.if.usp.br

Abstract

We consider the AdS space formulation of the classical dynamics deriving from the Stückelberg Lagrangian. The on-shell action is shown to be free of infrared singularities as the vector boson mass tends to zero. In this limit the model becomes Maxwell theory formulated in an arbitrary covariant gauge. Then we use the AdS/CFT correspondence to compute the two-point correlation functions on the boundary. It is shown that the gauge dependence concentrates on the contact terms while the non-trivial part of the conformal theory correlators turns out to be that already found by working in a completely fixed gauge.

PACS: 11.10.Kk, 11.25Mf

Keywords: AdS/CFT Correspondence, Maxwell theory, Gauge independence.

Typeset using REVTeX

As it is well known [1], Maldacena has conjectured that the large N limit of a certain conformal field theory (CFT) in a d -dimensional space can be described by string/M-theory on $AdS_{d+1} \times K$, where K is a suitable compact space. A precise form to this conjecture has been given in Refs. [2,3] according to which

$$Z_{AdS}[\phi_0] = \int_{\phi_0} \mathcal{D}\phi \exp(-I[\phi]) \equiv Z_{CFT}[\phi_0] = \langle \exp \left(\int_{\partial\Omega} d^d x \mathcal{O}\phi_0 \right) \rangle, \quad (1)$$

where ϕ_0 is the value taken by ϕ at the boundary. By assumption ϕ_0 is also the external current coupling to the operator \mathcal{O} in the boundary CFT. Thus the knowledge of the partition function in AdS_{d+1} enables one to obtain the correlation functions of the boundary CFT in d dimensions. The AdS/CFT correspondence has been studied for scalar fields [3–7], massive vector fields [8,9], spinor fields [8,10,11], the Rarita-Schwinger field [12–14], classical gravity [15,16], massive symmetric tensor fields [17], antisymmetric p -form fields [18,19], type IIB string theory [20,21] and three dimensional field theories with Chern-Simons terms [22].

The AdS/CFT correspondence is an example of the holographic principle [23] according to which a quantum theory with gravity must be describable by a boundary theory. This raises questions on how the detailed information of the theory in the bulk can be completely coded in a lower dimensional theory at the border. In fact this mechanism is still not well understood and several aspects of it have recently been investigated. For instance, the holographic bound, establishing that the boundary theory has only one bit of information per Planck area, manifests itself in the infrared-ultraviolet connection of the AdS/CFT correspondence [24]. Some situations involving superluminal oscillations and negative energy density have shown that there are hidden degrees of freedom which store information but have no local energy density [25]. On more conservative grounds, known relationships between field theories in the bulk should emerge in the conformal theory at the boundary. This has been verified explicitly for the case AdS_3/CFT_2 . In fact, the well known equivalence between Maxwell-Chern-Simons theory and the self-dual model in Minkowski space also holds in AdS_3 and, correspondingly, both models have been shown to lead to the same conformal theory at the border [22].

Another aspect of the holographic principle regards the gauge degrees of freedom of the bulk theory. If the bulk theory is a gauge theory the gauge degrees of freedom must be somehow codified at the border. Since the correlators of the corresponding conformal theory have conserved sources they do not carry information about de longitudinal modes of the gauge field. Apparently there is no information about the gauge degrees of freedom at the border.

For the Abelian gauge field the corresponding conformal correlators were found in [3] by evaluating the first term in the equality (1). The present work is dedicated to study the gauge dependence of this result. We shall demonstrate that the gauge degrees of freedom only contribute to the contact terms which appear in the on-shell action and are usually ignored when computing the correlators of the theory at the border.

Thus we shall be looking for a formulation of electrodynamics in an arbitrary gauge when the space-time background is AdS_{d+1} . As in the case of flat Minkowski space it will prove convenient to start from the Stückelberg action

$$I_S = - \int d^{d+1}x \sqrt{g} \left[\frac{1}{4} F_{\alpha\beta} g^{\alpha\rho} g^{\beta\sigma} F_{\rho\sigma} + \frac{m^2}{2} A_\alpha g^{\alpha\beta} A_\beta + \frac{1}{2a} (g^{\alpha\beta} \nabla_\alpha A_\beta)^2 \right], \quad (2)$$

where $F_{\alpha\beta} = \partial_\alpha A_\beta - \partial_\beta A_\alpha$, ∇_α is the covariant derivative and a is a real positive constant. Electrodynamics in an arbitrary covariant gauge, specified by the constant a , is defined as the limit $m^2 \rightarrow 0$ of Stückelberg theory. On the other hand the limit $a \rightarrow \infty$, while keeping $m^2 > 0$, results in the Proca theory. The mass term in (2) will help us to control the infrared divergent terms which will arise along the calculation.

As usual we take the representation of AdS_{d+1} in Poincaré coordinates which describes the half-space $x^0 > 0$, $x^i \in \mathbf{R}^d$ with the metric

$$g_{\mu\nu} = \frac{1}{(x^0)^2} \delta_{\mu\nu} . \quad (3)$$

The Lagrange equations of motion arising from (2) are found to read

$$\nabla_\mu F^{\mu\nu} + \frac{1}{a} \nabla_\mu L^{\mu\nu} - m^2 A^\nu = 0 , \quad (4)$$

where $L^{\mu\nu} \equiv g^{\mu\nu} g^{\alpha\beta} \nabla_\alpha A_\beta$.

Before going on a digression is in order. Since we are going to solve the equations of motion subjected to Dirichlet boundary conditions some care must be exercised when applying the variational principle to the action (2). When we vary the action to obtain the equations of motion the boundary term

$$- \int d^d x \sqrt{g} \left(F^{0i} \delta A_i + \frac{1}{a} \nabla^\mu A_\mu \delta A^0 \right) \Big|_{x^0=\epsilon} \quad (5)$$

is generated. If the gauge fixing term is not present then the boundary conditions must be prescribed only for the spatial components of the potential A_i . In the present case, however, all components of the potential must be given at the border. Then no additional boundary terms are needed in the original action to cancel the one coming from the variational principle.

After this clarification we return to our main line of development. The solving of the equations of motion is greatly simplified by the decomposition of A^ν into a scalar field Φ

$$\Phi \equiv \nabla_\nu A^\nu , \quad (6)$$

and a vector field U^ν

$$U^\nu \equiv A^\nu - \frac{1}{am^2} \nabla^\nu \Phi . \quad (7)$$

These new fields satisfy, respectively, the equations of motion

$$\left(g_{\mu\nu} \nabla^\mu \nabla^\nu - am^2 \right) \Phi = 0 , \quad (8)$$

and

$$\nabla_\mu U^{\mu\nu} - m^2 U^\nu = 0 , \quad (9)$$

as can be verified by using (4). Here $U_{\mu\nu} \equiv \partial_\mu U_\nu - \partial_\nu U_\mu$. Clearly U^μ is a Proca field with mass m since

$$\nabla_\mu U^\mu = 0 . \quad (10)$$

The solutions of the equations of motion (8) and (9), converging at $x^0 \rightarrow \infty$, have already been presented in the literature [4,8] and read, respectively,

$$\Phi = (x^0)^{\frac{d}{2}} \int \frac{d^d k}{(2\pi)^d} e^{-i\vec{k}\cdot\vec{x}} \phi(\vec{k}) K_{\alpha_a}(kx^0) , \quad (11)$$

$$\tilde{U}_0(x) = (x^0)^{\frac{d}{2}+1} \int \frac{d^d k}{(2\pi)^d} e^{-i\vec{k}\cdot\vec{x}} u_0(\vec{k}) K_{\tilde{\alpha}}(kx^0) , \quad (12a)$$

$$\tilde{U}_i(x) = (x^0)^{\frac{d}{2}} \int \frac{d^d k}{(2\pi)^d} e^{-i\vec{k}\cdot\vec{x}} \left[u_i(\vec{k}) K_{\tilde{\alpha}}(kx^0) + i u_0(\vec{k}) \frac{k_i}{k} x^0 K_{\tilde{\alpha}+1}(kx^0) \right] , \quad (12b)$$

where $x \equiv (x^0, \vec{x})$, $k \equiv |\vec{k}|$, K is the modified Bessel function and

$$\tilde{U}_\nu \equiv x^0 U_\nu \quad (13)$$

are the components of U with Lorentz indices [8]. Furthermore,

$$\alpha_a \equiv \sqrt{am^2 + \frac{d^2}{4}} , \quad (14a)$$

$$\tilde{\alpha} \equiv \sqrt{\frac{(d-2)^2}{4} + m^2} . \quad (14b)$$

We also recall that the Fourier transforms $u_0(\vec{k})$ and $u_i(\vec{k})$ are not all independent but related among themselves in order to secure the fulfillment of Eq.(10). It has been shown that [8]

$$u_0 = \frac{ikv}{\epsilon} \frac{K_{\tilde{\alpha}}(k\epsilon)}{K_{\tilde{\alpha}+1}(k\epsilon)} , \quad (15a)$$

$$u_i = v_i + v \frac{k_i}{k} , \quad (15b)$$

where

$$v = - \frac{k_i v_i}{k^2} \frac{k\epsilon K_{\tilde{\alpha}+1}(k\epsilon)}{\Sigma(k\epsilon)} , \quad (16)$$

$$\Sigma(k\epsilon) \equiv (\tilde{\Delta} - 1) K_{\tilde{\alpha}}(k\epsilon) + k\epsilon K_{\tilde{\alpha}-1}(k\epsilon) , \quad (17)$$

and

$$\tilde{\Delta} \equiv \tilde{\alpha} + \frac{d}{2} . \quad (18)$$

Here $x^0 = \epsilon > 0$ specifies a near-boundary surface.

As usual, we shall look for a solution written in terms of boundary field values specified at the near-boundary surface $x^0 = \epsilon$; the limit $\epsilon \rightarrow 0$ [5] being performed at the very end of the calculations. In particular, by returning with (11) and (12) into (7) one can determine

the unknowns ϕ and v_i in terms of the values assumed by \tilde{A}_μ on the surface $x^0 = \epsilon$. Thus, one arrives at the following final expressions for Φ and \tilde{U}_μ ,

$$\frac{1}{am^2} \Phi(x) = (x^0)^{\frac{d}{2}} (\epsilon)^{-\frac{d}{2}} \int \frac{d^d k}{(2\pi)^d} e^{-i\vec{k} \cdot \vec{x}} \left[\frac{W_j}{am^2} K_{\alpha_a}(kx^0) \tilde{A}_{\epsilon,j}(\vec{k}) + \frac{W}{am^2} K_{\alpha_a}(kx^0) \tilde{A}_{\epsilon,0}(\vec{k}) \right], \quad (19)$$

$$\tilde{U}_0(x) = (x^0)^{\frac{d}{2}+1} \epsilon^{-(\frac{d}{2}+1)} \int \frac{d^d k}{(2\pi)^d} e^{-i\vec{k} \cdot \vec{x}} \left[Z_j K_{\tilde{\alpha}}(kx^0) \tilde{A}_{\epsilon,j}(\vec{k}) - Z K_{\tilde{\alpha}}(kx^0) \tilde{A}_{\epsilon,0}(\vec{k}) \right], \quad (20)$$

and

$$\begin{aligned} \tilde{U}_i(x) = & (x^0)^{\frac{d}{2}} \epsilon^{-\frac{d}{2}} \int \frac{d^d k}{(2\pi)^d} e^{-i\vec{k} \cdot \vec{x}} \left\{ \left[X_{ij}^I K_{\tilde{\alpha}}(kx^0) + kx^0 X_{ij}^{II} K_{\tilde{\alpha}+1}(kx^0) \right] \tilde{A}_{\epsilon,j}(\vec{k}) \right. \\ & \left. - \left[Y_i^I K_{\tilde{\alpha}}(kx^0) + kx^0 Y_i^{II} K_{\tilde{\alpha}+1}(kx^0) \right] \tilde{A}_{\epsilon,0}(\vec{k}) \right\}, \end{aligned} \quad (21)$$

where

$$\frac{W_j}{am^2} = i\epsilon k_j \frac{K_{\tilde{\alpha}}(k\epsilon)}{\mathcal{D}(k\epsilon)}, \quad (22a)$$

$$\frac{W}{am^2} = \frac{\Sigma(k\epsilon)}{\mathcal{D}(k\epsilon)}, \quad (22b)$$

$$Z_j = -i\epsilon k_j \left[\frac{1}{\Sigma(k\epsilon)} - (k\epsilon)^2 \frac{K_{\tilde{\alpha}}(k\epsilon) K_{\alpha_a}(k\epsilon)}{\mathcal{D}(k\epsilon) \Sigma(k\epsilon)} \right], \quad (23a)$$

$$Z = -(k\epsilon)^2 \frac{K_{\alpha_a}(k\epsilon)}{\mathcal{D}(k\epsilon)}, \quad (23b)$$

$$X_{ij}^I = \frac{1}{K_{\tilde{\alpha}}(k\epsilon)} \left[\delta_{ij} - (k\epsilon) \frac{k_i k_j}{k^2} \frac{K_{\tilde{\alpha}+1}(k\epsilon)}{\Sigma(k\epsilon)} \right] + \left(\tilde{\alpha} + 1 - \frac{d}{2} \right) (\epsilon)^2 k_i k_j \frac{K_{\tilde{\alpha}}(k\epsilon) K_{\alpha_a}(k\epsilon)}{\mathcal{D}(k\epsilon) \Sigma(k\epsilon)}, \quad (24a)$$

$$X_{ij}^{II} = \frac{k_i k_j}{k^2} \frac{1}{\Sigma(k\epsilon)} - (\epsilon)^2 k_i k_j \frac{K_{\tilde{\alpha}}(k\epsilon) K_{\alpha_a}(k\epsilon)}{\mathcal{D}(k\epsilon) \Sigma(k\epsilon)}, \quad (24b)$$

$$Y_i^I = \left(\tilde{\alpha} + 1 - \frac{d}{2} \right) i\epsilon k_i \frac{K_{\alpha_a}(k\epsilon)}{\mathcal{D}(k\epsilon)}, \quad (25a)$$

$$Y_i^{II} = -i\epsilon k_i \frac{K_{\alpha_a}(k\epsilon)}{\mathcal{D}(k\epsilon)}, \quad (25b)$$

$$\mathcal{D}(k\epsilon) \equiv (k\epsilon)^2 K_{\tilde{\alpha}}(k\epsilon) K_{\alpha_a}(k\epsilon) + \Sigma(k\epsilon) \Lambda(k\epsilon), \quad (26)$$

and

$$\Lambda(k\epsilon) \equiv \left(\frac{d}{2} - \alpha_a \right) K_{\alpha_a}(k\epsilon) - k\epsilon K_{\alpha_a-1}(k\epsilon). \quad (27)$$

Furthermore,

$$\tilde{A}_{\epsilon,\mu}(\vec{k}) = \int d^d x e^{i\vec{k}\cdot\vec{x}} \tilde{A}_{\epsilon,\mu}(\vec{x}) , \quad (28)$$

where

$$\tilde{A}_{\epsilon,\mu}(\vec{x}) \equiv \tilde{A}_\mu(x^0 = \epsilon, \vec{x}) . \quad (29)$$

Notice that when $a \rightarrow \infty$ then $\Lambda \rightarrow \infty$ and, consequently, $\mathcal{D} \rightarrow \infty$. Hence,

$$\lim_{a \rightarrow \infty} X_{ij}^I = \frac{1}{K_{\tilde{\alpha}}(k\epsilon)} \left[\delta_{ij} - (k\epsilon) \frac{k_i k_j}{k^2} \frac{K_{\tilde{\alpha}+1}(ke)}{\Sigma(k\epsilon)} \right] , \quad (30a)$$

$$\lim_{a \rightarrow \infty} X_{ij}^{II} = \frac{k_i k_j}{k^2} \frac{1}{\Sigma(k\epsilon)} , \quad (30b)$$

$$\lim_{a \rightarrow \infty} Z_j = -i\epsilon k_j \frac{1}{\Sigma(k\epsilon)} , \quad (30c)$$

while

$$\lim_{a \rightarrow \infty} \frac{W_j}{am^2} = \lim_{a \rightarrow \infty} \frac{W}{am^2} = \lim_{a \rightarrow \infty} Z = \lim_{a \rightarrow \infty} Y_i^I = \lim_{a \rightarrow \infty} Y_i^{II} = 0 . \quad (31)$$

Correspondingly,

$$\lim_{a \rightarrow \infty} \frac{\Phi}{am^2} = 0 \implies \lim_{a \rightarrow \infty} A_\nu = U_\nu , \quad (32)$$

with

$$\tilde{U}_0(x) = -i\epsilon(x^0)^{\frac{d}{2}+1} \epsilon^{-(\frac{d}{2}+1)} \int \frac{d^d k}{(2\pi)^d} e^{-i\vec{k}\cdot\vec{x}} k_j \frac{1}{\Sigma(k\epsilon)} K_{\tilde{\alpha}}(kx^0) \tilde{U}_{\epsilon,j}(\vec{k}) , \quad (33a)$$

$$\begin{aligned} \tilde{U}_i(x) = & (x^0)^{\frac{d}{2}} \epsilon^{-\frac{d}{2}} \int \frac{d^d k}{(2\pi)^d} e^{-i\vec{k}\cdot\vec{x}} \left\{ \frac{1}{K_{\tilde{\alpha}}(k\epsilon)} \left[\delta_{ij} - (k\epsilon) \frac{k_i k_j}{k^2} \frac{K_{\tilde{\alpha}+1}(ke)}{\Sigma(k\epsilon)} \right] K_{\tilde{\alpha}}(kx^0) \right. \\ & \left. + kx^0 \frac{k_i k_j}{k^2} \frac{1}{\Sigma(k\epsilon)} K_{\tilde{\alpha}+1}(kx^0) \right\} \tilde{U}_{\epsilon,j}(\vec{k}) , \end{aligned} \quad (33b)$$

which, as expected, reproduce the results found in Ref. [8] for the Proca field.

In this paper we are interested in Maxwell theory formulated in an arbitrary covariant gauge. Therefore we must investigate the limit $m^2 \rightarrow 0$ of (19–25). From Eqs.(14a) and (14b) one finds, respectively, that the expansions

$$\tilde{\alpha} = \frac{d}{2} - 1 + \tilde{\eta}m^2 , \quad (34a)$$

$$\alpha_a = \frac{d}{2} + \eta_a m^2 , \quad (34b)$$

with

$$\tilde{\eta} = \frac{1}{d-2} , \quad (35a)$$

$$\eta_a = \frac{a}{d} , \quad (35b)$$

are valid up to terms of order m^2 . Then from Eqs.(17), (26) and (27) it follows that $\mathcal{D}(k\epsilon)$ vanishes as m^2 goes to zero. In fact, it is straightforward to show that

$$\begin{aligned} \lim_{m^2 \rightarrow 0} \mathcal{D}(k\epsilon) \rightarrow m^2 \left\{ (k\epsilon)^2 (\eta_a - \tilde{\eta}) \left[K_{\frac{d}{2}-1}(k\epsilon) \frac{\partial K_\rho(k\epsilon)}{\partial \rho} \Big|_{\rho=\frac{d}{2}} - K_{\frac{d}{2}}(k\epsilon) \frac{\partial K_\rho(k\epsilon)}{\partial \rho} \Big|_{\rho=\frac{d}{2}-1} \right] \right. \\ \left. + (k\epsilon) \tilde{\eta} K_{\frac{d}{2}-1}(k\epsilon) K_{\frac{d}{2}-1}(k\epsilon) - (k\epsilon) \eta_a K_{\frac{d}{2}}(k\epsilon) K_{\frac{d}{2}}(k\epsilon) \right\} . \end{aligned} \quad (36)$$

On the other hand, (34a) implies that

$$\lim_{m^2 \rightarrow 0} \left(\tilde{\alpha} + 1 - \frac{d}{2} \right) = m^2 \tilde{\eta} , \quad (37)$$

which also vanishes as $m^2 \rightarrow 0$. Therefore, W_j/am^2 , W/am^2 , Z_j , Z , X_{ij}^{II} and Y_i^{II} become singular as $m^2 \rightarrow 0$, while X_{ij}^I and Y_i^I remain finite (see Eqs.(22-25)). This implies that Φ/am^2 and U_ν develop infrared divergences. The question is now whether A_ν is well defined in the zero mass limit. To investigate this point we go back with Eqs.(19-21) into Eq.(7) obtaining

$$\begin{aligned} \tilde{A}_0(x) = (x^0)^{\frac{d}{2}} (\epsilon)^{-\frac{d}{2}} \int \frac{d^d k}{(2\pi)^d} e^{-i\vec{k} \cdot \vec{x}} \left\{ \left[-\frac{x^0}{\epsilon} \frac{i\epsilon k_j}{\Sigma(k\epsilon)} K_{\tilde{\alpha}}(kx^0) \right. \right. \\ \left. \left. + \frac{x^0}{\epsilon} i\epsilon k_j (k\epsilon)^2 \frac{K_{\tilde{\alpha}}(k\epsilon) K_{\alpha_a}(k\epsilon)}{\mathcal{D}(k\epsilon) \Sigma(k\epsilon)} K_{\tilde{\alpha}}(kx^0) + \frac{d}{2} \frac{W_j}{am^2} K_{\alpha_a}(kx^0) + \frac{W_j}{am^2} kx^0 \frac{\partial K_{\alpha_a}(kx^0)}{\partial(kx^0)} \right] \tilde{A}_{\epsilon,j}(\vec{k}) \right. \\ \left. + \left[\frac{d}{2} \frac{W}{am^2} K_{\alpha_a}(kx^0) + \frac{W}{am^2} kx^0 \frac{\partial K_{\alpha_a}(kx^0)}{\partial(kx^0)} - \frac{x^0}{\epsilon} Z K_{\tilde{\alpha}}(kx^0) \right] \tilde{A}_{\epsilon,0}(\vec{k}) \right\} \end{aligned} \quad (38)$$

and

$$\begin{aligned} \tilde{A}_i(x) = (x^0)^{\frac{d}{2}} (\epsilon)^{-\frac{d}{2}} \int \frac{d^d k}{(2\pi)^d} e^{-i\vec{k} \cdot \vec{x}} \left\{ \left[X_{ij}^I K_{\tilde{\alpha}}(kx^0) + kx^0 \frac{k_i k_j}{k^2} \frac{1}{\Sigma(k\epsilon)} K_{\tilde{\alpha}+1}(kx^0) \right. \right. \\ \left. \left. - kx^0 (\epsilon)^2 k_i k_j \frac{K_{\tilde{\alpha}}(k\epsilon) K_{\alpha_a}(k\epsilon)}{\mathcal{D}(k\epsilon) \Sigma(k\epsilon)} K_{\tilde{\alpha}+1}(kx^0) - ix^0 k_i \frac{W_j}{am^2} K_{\alpha_a}(kx^0) \right] \tilde{A}_{\epsilon,j}(\vec{k}) \right. \\ \left. - \left[Y_i^I K_{\tilde{\alpha}}(kx^0) + kx^0 Y_i^{II} K_{\tilde{\alpha}+1}(kx^0) + ix^0 k_i \frac{W}{am^2} K_{\alpha_a}(kx^0) \right] \tilde{A}_{\epsilon,0}(\vec{k}) \right\} . \end{aligned} \quad (39)$$

The three terms in the second line of (38) are, individually, infrared divergent. However, after adding them up one finds

$$\begin{aligned} & \frac{x^0}{\epsilon} i\epsilon k_j (k\epsilon)^2 \frac{K_{\tilde{\alpha}}(k\epsilon) K_{\alpha_a}(k\epsilon)}{\mathcal{D}(k\epsilon) \Sigma(k\epsilon)} K_{\tilde{\alpha}}(kx^0) + \frac{d}{2} \frac{W_j}{am^2} K_{\alpha_a}(kx^0) + \frac{W_j}{am^2} kx^0 \frac{\partial K_{\alpha_a}(kx^0)}{\partial(kx^0)} \\ &= i\epsilon k_j \frac{K_{\tilde{\alpha}}(k\epsilon)}{\mathcal{D}(k\epsilon)} \left\{ \left(\frac{d}{2} - \alpha_a \right) K_{\alpha_a}(kx^0) + kx^0 \left[k\epsilon \frac{K_{\alpha_a}(k\epsilon)}{\Sigma(k\epsilon)} K_{\tilde{\alpha}}(kx^0) - K_{\alpha_a-1}(kx^0) \right] \right\} \\ &= i\epsilon k_j \frac{K_{\tilde{\alpha}}(k\epsilon)}{\mathcal{D}(k\epsilon)} \times O(m^2) , \end{aligned} \quad (40)$$

where $O(m^2)$ denotes terms of order m^2 or higher. Clearly, $O(m^2)$ cancels out the divergence arising from $1/\mathcal{D}(k\epsilon)$ (see Eq.(36)) leaving us with a finite expression. For arriving at this result we have used the fact that for small m^2 ,

$$K_{\alpha_a} = K_{\frac{d}{2}} + O(m^2) , \quad (41a)$$

$$K_{\tilde{\alpha}+1} = K_{\frac{d}{2}} + O(m^2) , \quad (41b)$$

$$\Sigma(k\epsilon) = k\epsilon K_{\frac{d}{2}}(k\epsilon) + O(m^2) , \quad (41c)$$

$$\Lambda(k\epsilon) = -k\epsilon K_{\frac{d}{2}-1}(k\epsilon) + O(m^2) , \quad (41d)$$

which can easily be corroborated (see Eqs.(17) and (34)). Through a similar analysis we show that the third line in Eq.(38) and the second and third lines in Eq.(39) define functions of m^2 which are regular at $m^2 = 0$. To summarize, $A_\nu(x)$ is indeed an analytic function of m^2 in the vicinity of $m^2 = 0$.

We turn next into application of the AdS/CFT correspondence to compute the two-point correlation function $\langle \mathcal{O}_\mu(\vec{x}) \mathcal{O}_\nu(\vec{y}) \rangle$ of the boundary CFT. The dominant term in the path integral in Eq.(1) is the exponential of the classical action evaluated on-shell. After using Eqs.(2) and (4) one finds that

$$I_S = \frac{1}{2} \int_{\partial\Omega} d^d x \epsilon^{-d} \left\{ \tilde{A}_{\epsilon,i}(\vec{x}) \left[-\tilde{A}_{\epsilon,i}(\vec{x}) + \epsilon \tilde{F}_{\epsilon,0i}(\vec{x}) \right] + \frac{1}{a} \tilde{A}_{\epsilon,0}(\vec{x}) \left[-d \tilde{A}_{\epsilon,0}(\vec{x}) + \epsilon \left(\frac{\partial \tilde{A}_0(x)}{\partial x^0} \Big|_{x^0=\epsilon} + \partial_i \tilde{A}_{\epsilon,i} \right) \right] \right\} , \quad (42)$$

where $\tilde{F}_{0i}(x) \equiv \partial_0 \tilde{A}_i(x) - \partial_i \tilde{A}_0(x)$ and $\partial\Omega$ is the near-boundary surface $x^0 = \epsilon$. The explicit dependence of I_S on the gauge parameter a should be noticed. Non trivial field theory correlators may only arise from those terms containing $\tilde{F}_{\epsilon,0i}(\vec{x})$ and $\partial \tilde{A}_0(x)/\partial x^0|_{x^0=\epsilon}$. We shall therefore concentrate our attention on these objects.

From (38) and (39) and after setting $x^0 = \epsilon$ one obtains

$$\begin{aligned} \tilde{F}_{\epsilon,0i}(\vec{x}) &= \left(\frac{d}{2} - \tilde{\alpha} \right) \frac{1}{\epsilon} \tilde{A}_{\epsilon,i}(\vec{x}) \\ &+ \int \frac{d^d k}{(2\pi)^d} e^{-i\vec{k}\cdot\vec{x}} \left[\frac{k K_{\tilde{\alpha}-1}}{K_{\tilde{\alpha}}} \left(-\delta_{ij} + \frac{k_i k_j}{k^2} k\epsilon \frac{K_{\tilde{\alpha}+1}}{\Sigma} \right) \right] \tilde{A}_{\epsilon,j}(\vec{k}) \\ &+ (\tilde{\Delta} - 1) \left(\tilde{\alpha} + 1 - \frac{d}{2} \right) \int \frac{d^d k}{(2\pi)^d} e^{-i\vec{k}\cdot\vec{x}} \left[\epsilon k_i k_j \frac{K_{\alpha_a} K_{\tilde{\alpha}}^2}{\Sigma \mathcal{D}} \tilde{A}_{\epsilon,j}(\vec{k}) - i k_i \frac{K_{\alpha_a} K_{\tilde{\alpha}}}{\mathcal{D}} \tilde{A}_{\epsilon,0}(\vec{k}) \right] , \end{aligned} \quad (43)$$

where the argument $k\epsilon$ of the modified Bessel functions has been omitted in order to simplify the notation. The first two lines in the right hand side of (43) survive in the limit $a \rightarrow \infty$ and reproduce, as it must be the case, the result for the Proca field tensor [8]. Furthermore the dangerous infrared behavior of \mathcal{D} , showing up in the denominator of the last line of (43), is again canceled by the factor $\tilde{\alpha} + 1 - d/2$ in the corresponding numerator, leaving us with an overall function $\tilde{F}_{\epsilon,0i}(\vec{x})$ regular at $m^2 = 0$.

As for $\partial \tilde{A}_0(x)/\partial x^0|_{x^0=\epsilon}$ Eq.(38) leads to

$$\begin{aligned} \frac{\partial \tilde{A}_0(x)}{\partial x^0} \Big|_{x^0=\epsilon} &= \left(\frac{d}{2} + 1 \right) \frac{1}{\epsilon} \tilde{A}_{\epsilon,0}(\vec{x}) + \frac{1}{\epsilon} \int \frac{d^d k}{(2\pi)^d} e^{-i\vec{k}\cdot\vec{x}} \\ &\times \left\{ \left[1 + \left(\alpha_a - \frac{d}{2} \right)^2 \frac{K_{\alpha_a} K_{\tilde{\alpha}}}{\mathcal{D}} \right] \tilde{A}_{\epsilon,j}(\vec{k}) + \left[\left(\frac{d}{2} - 1 \right) + \left(\alpha_a^2 - \frac{d^2}{4} \right) \frac{\Sigma K_{\alpha_a}}{\mathcal{D}} \right] \tilde{A}_{\epsilon,0}(\vec{k}) \right\}. \end{aligned} \quad (44)$$

Since $(\alpha_a - \frac{d}{2})$ and \mathcal{D} are $O(m^2)$ the second term in the first bracket of Eq.(44) is $O(m^2)$ and, therefore, it drops out in the zero mass limit. For the same reasons the term involving $1/\mathcal{D}$ in the second bracket of (44) is regular in $m^2 = 0$ which renders $\partial \tilde{A}_0(x)/\partial x^0|_{x^0=\epsilon}$ free of infrared divergences.

We shall next determine the contributions of each term in (43) and (44) to the action in (42) and, therefore, to the correlators $\langle \mathcal{O}_\mu(\vec{x}) \mathcal{O}_\nu(\vec{y}) \rangle$. We shall do this for $m^2 > 0$, the limit $m^2 \rightarrow 0$ being taken at the very end of the calculations.

Clearly the first line in the right hand side of (43) contributes with a contact term

$$I_S^{(1)} = \frac{1}{2} \left(\frac{d}{2} - \tilde{\alpha} \right) \epsilon^{(2-d)} \int d^d x d^d y A_{\epsilon,i}(\vec{x}) \delta(\vec{x} - \vec{y}) A_{\epsilon,i}(\vec{y}), \quad (45)$$

where we have restored the original field variables by replacing $\tilde{A}_{\epsilon,i}(\vec{x}) = \epsilon A_{\epsilon,i}(\vec{x})$ (see Eq.(13)).

The second line in the right hand side of (43), to be denoted by $I_i^{(2)}(\vec{x})$, can be cast as

$$I_i^{(2)}(\vec{x}) = \int d^d y \tilde{A}_{\epsilon,j}(\vec{y}) I_{ij}^{(2)}(\vec{x}, \vec{y}), \quad (46)$$

where

$$I_{ij}^{(2)}(\vec{x}, \vec{y}) \equiv \int \frac{d^d k}{(2\pi)^d} e^{-i\vec{k}\cdot(\vec{x}-\vec{y})} \left[\frac{k K_{\tilde{\alpha}-1}}{K_{\tilde{\alpha}}} \left(-\delta_{ij} + \frac{k_i k_j}{k^2} k \epsilon \frac{K_{\tilde{\alpha}+1}}{\Sigma} \right) \right]. \quad (47)$$

The calculation of $I_{ij}^{(2)}$ was already carried out in Ref. [8] and yields

$$I_i^{(2)}(\vec{x}) = 2 \frac{\tilde{\Delta}}{\tilde{\Delta} - 1} \tilde{\alpha} \tilde{c} \epsilon^{2\tilde{\alpha}-1} \int d^d y \tilde{A}_{\epsilon,j}(\vec{y}) \frac{1}{|\vec{x} - \vec{y}|^{2\tilde{\Delta}}} \left[\delta_{ij} - 2 \frac{(x_i - y_i)(x_j - y_j)}{|\vec{x} - \vec{y}|^2} \right], \quad (48)$$

where

$$\tilde{c} = \frac{\Gamma(\tilde{\Delta})}{\pi^{\frac{d}{2}} \Gamma(\tilde{\alpha})}. \quad (49)$$

The corresponding contribution to the action in Eq.(42) is then given by

$$\begin{aligned} I_S^{(2)} &= \frac{\tilde{\Delta}}{\tilde{\Delta} - 1} \tilde{\alpha} \tilde{c} \epsilon^{2(\tilde{\Delta}-d)} \\ &\times \int d^d x d^d y \tilde{A}_{\epsilon,i}(\vec{x}) \tilde{A}_{\epsilon,j}(\vec{y}) \frac{1}{|\vec{x} - \vec{y}|^{2\tilde{\Delta}}} \left[\delta_{ij} - 2 \frac{(x_i - y_i)(x_j - y_j)}{|\vec{x} - \vec{y}|^2} \right]. \end{aligned} \quad (50)$$

A nontrivial contribution to the correlation function of the boundary CFT arises from the partial action $I_S^{(2)}$. To find it we start by recalling that the external source for the conformal

field \mathcal{O}_μ is in the present case $A_{0,\mu}(\vec{x}) \equiv A_\mu(x^0 = 0, \vec{x})$. Therefore we must perform the limit $\epsilon \rightarrow 0$ in (50). However such limit only becomes operational after prescribing a relationship relating $A_{\epsilon,\mu}(\vec{x})$ with the value of the field on the boundary surface $x^0 = 0$. We then identify

$$A_{0,\mu}(\vec{x}) = \lim_{\epsilon \rightarrow 0} \epsilon^{\tilde{\Delta}-d} \tilde{A}_{\epsilon,\mu}(\vec{x}) . \quad (51)$$

Notice that $\tilde{\Delta} - d$ is gauge independent. Hence, at the limit $a \rightarrow \infty$ Eqs.(51) and (32) lead to

$$U_{0,\mu}(\vec{x}) = \lim_{\epsilon \rightarrow 0} \epsilon^{\tilde{\Delta}-d} \tilde{U}_{\epsilon,\mu}(\vec{x}) , \quad (52)$$

which, as expected, reproduces the normalization prescription for the Proca field [8]. Then by taking into account the normalization in Eq.(51) and using the AdS/CFT correspondence (see Eq.(1)) one reads off from Eq.(50) the two-point correlators

$$\langle \mathcal{O}_i(\vec{x}) \mathcal{O}_j(\vec{y}) \rangle \Big|_{m^2=0} = \text{Constant} \times \frac{1}{|\vec{x} - \vec{y}|^{2d-2}} \left[\delta_{ij} - 2 \frac{(x_i - y_i)(x_j - y_j)}{|\vec{x} - \vec{y}|^2} \right] , \quad (53)$$

in agreement with the results already obtained for this object in the case of the Abelian gauge field [3,5].

The first k -integral in the third line of Eq.(43) can be written

$$I_i^{(31)}(\vec{x}) \equiv - \frac{\partial^2}{\partial x^i \partial x^j} \int d^d y \tilde{A}_{\epsilon,j}(\vec{y}) I^{(31)}(\vec{x}, \vec{y}) , \quad (54)$$

with

$$I^{(31)}(\vec{x}, \vec{y}) = (\tilde{\Delta} - 1) \left(\tilde{\alpha} + 1 - \frac{d}{2} \right) \int \frac{d^d k}{(2\pi)^d} e^{-i\vec{k} \cdot (\vec{x} - \vec{y})} \epsilon \frac{K_{\alpha_a} K_{\tilde{\alpha}}^2}{\Sigma \mathcal{D}} . \quad (55)$$

We are required to compute $I^{(31)}$ in the limit $\epsilon \rightarrow 0$. As in Refs. [4,8] we replace the modified Bessel functions appearing in the integrand of (55) by the first terms in the series expansion

$$K_\alpha(k\epsilon) = \left(\frac{k\epsilon}{2} \right)^{-\alpha} \frac{\Gamma(\alpha)}{2} \left[1 - \left(\frac{k\epsilon}{2} \right)^{2\alpha} D_0(\alpha) + \dots \right] . \quad (56)$$

where the dots indicate terms of order $(k\epsilon/2)^{2n}$ and $(k\epsilon/2)^{2\alpha+2n}$. The leading terms in the expansion of the numerator turn out to be

$$\begin{aligned} \epsilon K_{\alpha_a} K_{\tilde{\alpha}}^2 &= \frac{\Gamma(\alpha_a) \Gamma^2(\tilde{\alpha})}{4k} \left(\frac{k\epsilon}{2} \right)^{-(2\tilde{\alpha} + \alpha_a)} \\ &\times \left[\left(\frac{k\epsilon}{2} \right) - 2 \left(\frac{k\epsilon}{2} \right)^{2\tilde{\alpha}+1} D_0(\tilde{\alpha}) - \left(\frac{k\epsilon}{2} \right)^{2\alpha_a+1} D_0(\alpha_a) \right] , \end{aligned} \quad (57)$$

while for the denominator one finds, after some algebra,

$$\begin{aligned} \Sigma \mathcal{D} &= (\tilde{\Delta} - 1)^2 \left(\frac{d}{2} - \alpha_a \right) \frac{\Gamma(\alpha_a) \Gamma^2(\tilde{\alpha})}{8} \left(\frac{k\epsilon}{2} \right)^{-(2\tilde{\alpha} + \alpha_a)} \\ &\times \left[1 - 2 \left(\frac{k\epsilon}{2} \right)^{2\tilde{\alpha}} D_0(\tilde{\alpha}) E_0(\tilde{\alpha}, \alpha_a) - \left(\frac{k\epsilon}{2} \right)^{2\alpha_a - 1} D_0(\alpha_a) F_0(\tilde{\alpha}, \alpha_a) \right] . \end{aligned} \quad (58)$$

Here,

$$D_0(\alpha) \equiv \frac{\Gamma(1 - \alpha)}{\Gamma(1 + \alpha)} , \quad (59a)$$

$$E_0(\tilde{\alpha}, \alpha_a) = 1 - 2 \frac{\tilde{\alpha}}{(\tilde{\Delta} - 1)} \tilde{\alpha} D_0(\tilde{\alpha} - 1) \left[1 - 2 \frac{\alpha_a}{\left(\frac{d}{2} - \alpha_a\right)} D_0(\alpha_a) \right] , \quad (59b)$$

$$F_0(\tilde{\alpha}, \alpha_a) = -2 \frac{\alpha_a}{\left(\frac{d}{2} - \alpha_a\right)} D_0(\alpha_a - 1) , \quad (59c)$$

together with Eqs.(17), (26) and (27) have been used. As a consequence, on the near boundary-surface,

$$\begin{aligned} \epsilon \frac{K_{\alpha_a} K_{\tilde{\alpha}}^2}{\Sigma \mathcal{D}} &= \frac{2}{(\tilde{\Delta} - 1)^2 \left(\frac{d}{2} - \alpha_a\right)} \\ &\times \frac{1}{k} \left\{ \frac{k\epsilon}{2} + 2 \left(\frac{k\epsilon}{2} \right)^{2\tilde{\alpha} + 1} D_0(\tilde{\alpha}) [E_0(\tilde{\alpha}, \alpha_a) - 1] + \left(\frac{k\epsilon}{2} \right)^{2\alpha_a} D_0(\alpha_a) F_0(\tilde{\alpha}, \alpha_a) \right\} . \end{aligned} \quad (60)$$

At the level of the action the leading powers of ϵ are, correspondingly, $\epsilon^{2\tilde{\alpha} + 2 - d}$ and $\epsilon^{2\alpha_a + 1 - d}$, as seen from (42). By its turn the two-point CFT correlator incorporates the extra power $\epsilon^{-2(\tilde{\Delta} - d)}$ arising from (51) and its leading terms, other than contact terms, turn out to be either proportional to $\epsilon^{2\tilde{\alpha} + 2 - d - 2(\tilde{\Delta} - d)}$ or to $\epsilon^{2\alpha_a + 1 - d - 2(\tilde{\Delta} - d)}$. After taking into account Eq.(18) one obtains for the gauge independent power

$$\lim_{\epsilon \rightarrow 0} \epsilon^{2\tilde{\alpha} + 2 - d - 2(\tilde{\Delta} - d)} = \lim_{\epsilon \rightarrow 0} \epsilon^2 = 0 , \quad (61)$$

while for the gauge dependent one Eqs.(14) and (18) lead to

$$\lim_{\epsilon \rightarrow 0} \epsilon^{2\alpha_a + 1 - d - 2(\tilde{\Delta} - d)} = \lim_{\epsilon \rightarrow 0} \epsilon^{3 + O(m^2)} = 0 , \quad (62)$$

for any value of a .

As for the piece of the correlator $\langle \mathcal{O}_\mu(\vec{x}) \mathcal{O}_\nu(\vec{y}) \rangle$ originating in the second k -integral in the third line of Eq.(43), i.e.,

$$I_i^{(32)} \equiv (\tilde{\Delta} - 1) \left(\tilde{\alpha} + 1 - \frac{d}{2} \right) \frac{\partial}{\partial x^i} \int d^d y \tilde{A}_{\epsilon,0}(\vec{y}) \left[\int \frac{d^d k}{(2\pi)^d} e^{-i\vec{k} \cdot (\vec{x} - \vec{y})} \frac{K_{\alpha_a} K_{\tilde{\alpha}}}{\mathcal{D}} \right] , \quad (63)$$

a similar analysis reveals that this time the leading terms are either proportional to $\epsilon^{2\tilde{\alpha} + 1 - d - 2(\tilde{\Delta} - d)}$ or to $\epsilon^{2\alpha_a + 1 - d - 2(\tilde{\Delta} - d)}$. The gauge dependent power vanishes again as indicated in (62) while the gauge independent one vanishes linearly with ϵ . Indeed,

$$\lim_{\epsilon \rightarrow 0} \epsilon^{2\tilde{\alpha}+1-d-2(\tilde{\Delta}-d)} = \lim_{\epsilon \rightarrow 0} \epsilon^1 = 0 , \quad (64)$$

as implied by Eq.(18).

Summarizing, the third line in the right hand side of Eq.(43) only makes trivial contributions to the two-point CFT correlator. Thus if $\langle \mathcal{O}_\mu(\vec{x}) \mathcal{O}_\nu(\vec{y}) \rangle$ contains at all a nontrivial gauge dependent piece it can only originate in the last term of the right hand side of Eq.(44) which, as we already said, survives in the limit $m^2 \rightarrow 0$. We now turn into the analysis of such term, hereafter to be referred to as $I^{(0)}$, i.e.,

$$I^{(0)}(\vec{x}) = \int d^d y \tilde{A}_{\epsilon,0}(\vec{y}) I^{(00)}(\vec{x}, \vec{y}) \quad (65)$$

where

$$I^{(00)}(\vec{x}, \vec{y}) = \left(\alpha_a^2 - \frac{d^2}{4} \right) \int \frac{d^d k}{(2\pi)^d} e^{-i\vec{k} \cdot (\vec{x} - \vec{y})} \frac{1}{\epsilon} \frac{\Sigma K_{\alpha_a}}{\mathcal{D}} . \quad (66)$$

By using Eqs.(17), (26), (27) and (56) it can be verified that at the limit $\epsilon \rightarrow 0$ the following expansions hold

$$\begin{aligned} \Sigma K_{\alpha_a} &= (\tilde{\Delta} - 1) \frac{\Gamma(\alpha_a) \Gamma(\tilde{\alpha})}{4} \left(\frac{k\epsilon}{2} \right)^{-(\tilde{\alpha} + \alpha_a)} \\ &\times \left[1 - \left(\frac{k\epsilon}{2} \right)^{2\tilde{\alpha}} G_0(\tilde{\alpha}) - \left(\frac{k\epsilon}{2} \right)^{2\alpha_a} D_0(\alpha_a) \right] , \end{aligned} \quad (67)$$

$$\begin{aligned} \mathcal{D} &= (\tilde{\Delta} - 1) \left(\frac{d}{2} - \alpha_a \right) \frac{\Gamma(\alpha_a) \Gamma(\tilde{\alpha})}{4} \left(\frac{k\epsilon}{2} \right)^{-(\tilde{\alpha} + \alpha_a)} \\ &\times \left[1 - \left(\frac{k\epsilon}{2} \right)^{2\tilde{\alpha}} G_0(\tilde{\alpha}) - \left(\frac{k\epsilon}{2} \right)^{2\alpha_a} H_0(\alpha_a) \right] , \end{aligned} \quad (68)$$

where

$$G_0(\tilde{\alpha}) \equiv D_0(\tilde{\alpha}) \left[1 + 2 \frac{\tilde{\alpha}}{(\tilde{\Delta} - 1)} D_0(\tilde{\alpha} - 1) \right] , \quad (69a)$$

$$H_0(\tilde{\alpha}) \equiv D_0(\alpha_a) \left[1 + 2 \frac{\alpha_a}{(\frac{d}{2} - \alpha_a)} D_0(\alpha_a - 1) \right] . \quad (69b)$$

We emphasize a common feature of the expansions (67) and (68), namely, both of them have $-G_0$ as the coefficient of the power $\epsilon^{2\tilde{\alpha}}$. This is at the root of the absence of gauge dependent terms in $\langle \mathcal{O}_\mu(\vec{x}) \mathcal{O}_\nu(\vec{y}) \rangle$ because it implies that

$$\frac{1}{\epsilon} \frac{\Sigma K_{\alpha_a}}{\mathcal{D}} = \frac{1}{(\frac{d}{2} - \alpha_a)} \frac{1}{\epsilon} \left[1 - \left(\frac{k\epsilon}{2} \right)^{2\alpha_a} 2 \frac{\alpha_a}{(\frac{d}{2} - \alpha_a)} D_0(\alpha_a) D_0(\alpha_a - 1) + \mathcal{O}(\epsilon^{2\tilde{\alpha}+2}) \right] . \quad (70)$$

In words, all *potentially dangerous* powers $\epsilon^{2\tilde{\alpha}-1}$ cancel out among themselves leaving $\epsilon^{2\tilde{\alpha}+1}$ and $\epsilon^{2\alpha_a}$ as the leading powers in the expansion of the integrand in Eq.(66). As indicated in Eqs.(61) and (62) these powers do not contribute to $\langle \mathcal{O}_\mu(\vec{x})\mathcal{O}_\nu(\vec{y}) \rangle$ in the limit $\epsilon \rightarrow 0$.

We then conclude that the gauge dependence concentrates on the contact terms while the non-trivial part of the boundary conformal theory correlators turns out to be that already found by working in a completely fixed gauge [3,5] and displayed in Eq.(53). Another important feature is that although we have fixed all components of the potential at the border the pieces containing $\tilde{A}_{\epsilon,0}$ give only contact terms and the only non-trivial pieces are those containing $\tilde{A}_{\epsilon,i}$. Therefore the boundary theory still retains information on the gauge degrees of freedom of the bulk theory. This then lends further support to the holographic principle.

REFERENCES

- * Supported in part by Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq) and Fundação de Amparo à Pesquisa do Estado de São Paulo (FAPESP).
- [1] J. Maldacena, “The Large N Limit of Superconformal Field Theories and Supergravity”, hep-th/9711200, Adv. Theor. Math. Phys. **2** (1998) 231.
 - [2] S. Gubser, I. Klebanov and A. Polyakov, “Gauge Theory Correlators from Non-Critical String Theory”, hep-th/9802109, Phys. Lett. **428** (1998) 105.
 - [3] E. Witten, “Anti-de Sitter Space and Holography”, hep-th/9802150, Adv. Theor. Math. Phys. **2** (1998) 253.
 - [4] W. Mück and K. S. Viswanathan, “Conformal Field Theory Correlators from Classical Scalar Field Theory on AdS_{d+1} ”, hep-th/9804035, Phys. Rev. **D58** (1998) 41901.
 - [5] D. Z. Freedman, S. D. Mathur, A. Matusis and L. Rastelli, “Correlation Functions in the CFT_d/AdS_{d+1} Correspondence”, hep-th/9804058, Nucl. Phys. **B546** (1999) 96.
 - [6] V. Balasubramanian, P. Kraus and A. Lawrence, “Bulk vs. Boundary Dynamics in Anti-de Sitter Spacetime”, hep-th/9805171, Phys. Rev. **D59** (1999) 046003.
 - [7] P. Minces and V. O. Rivelles, “Scalar Field Theory in the AdS/CFT Correspondence Revisited”, hep-th/9907079.
 - [8] W. Mück and K. S. Viswanathan, “Conformal Field Theory Correlators from Classical Field Theory on Anti-de Sitter Space II. Vector and Spinor Fields”, hep-th/9805145, Phys. Rev. **D58** (1998) 106006.
 - [9] W. S. l’Yi, “Coordinate-Space Holographic Projection of Fields and an Application to Massive Vector Fields”, hep-th/9808051.
 - [10] M. Henningson and K. Sfetsos, “Spinors and the AdS/CFT Correspondence”, hep-th/9803251, Phys. Lett. **B431** (1998) 63.
 - [11] A. Ghezelbash, K. K. Kaviani, S. Parvizi and A. Fatollahi, “Interacting Spinors-Scalars and AdS/CFT Correspondence”, hep-th/9805162, Phys. Lett. **B435** (1998) 291.
 - [12] A. Volovich, “Rarita-Schwinger Field in the AdS/CFT Correspondence”, hep-th/9809009, J. High. En. Phys. **9809** (1998) 22.
 - [13] A. Koshelev and O. Rytchkov, “Note on the Massive Rarita-Schwinger Field in the AdS/CFT correspondence”, hep-th/9812238, Phys. Lett. **B450** (1999) 368.
 - [14] P. Matlock and K. S. Viswanathan, “The AdS/CFT Correspondence for the Massive Rarita-Schwinger Field”, hep-th/9906077.
 - [15] H. Liu and A. Tseytlin, “D=4 Super Yang-Mills, D=5 Gauged Supergravity and D=4 Conformal Supergravity”, hep-th/9804083, Nucl. Phys. **B533** (1998) 88.
 - [16] W. Mück and K. S. Viswanathan, “The Graviton in the AdS-CFT Correspondence: Solution Via the Dirichlet Boundary Value Problem”, hep-th/9810151.
 - [17] A. Polishchuk, “Massive Symmetric Tensor Field on AdS”, hep-th/9905048, J. High En. Phys. **9907** (1999) 007.
 - [18] G. Arutyunov and S. Frolov, “Antisymmetric Tensor Field on AdS_5 ”, hep-th/9807046, Phys. Lett. **B441** (1998) 173.
 - [19] W. l’Yi, “Correlators of Currents Corresponding to the Massive p -form Fields in AdS/CFT Correspondence”, hep-th/9809132, Phys. Lett. **B448** (1999) 218.
 - [20] T. Banks and M. Green, “Non-perturbative Effects in $AdS_5 \times S^5$ String Theory and d=4 SUSY Yang-Mills”, hep-th/9804170, J. High. En. Phys. **9805** (1998) 002.

- [21] G. Chalmers, H. Nastase, K. Schalm and R. Siebelink, “R-Current Correlators in N=4 Super Yang-Mills Theory from Anti-de Sitter Supergravity”, hep-th/9805105.
- [22] P. Minces and V. O. Rivelles, “Chern-Simons Theories in the AdS/CFT Correspondence”, hep-th/9902123, Phys. Lett. **B455** (1999) 147.
- [23] G. 'tHooft, “Dimensional Reduction in Quantum Gravity”, gr-qc/9310026; L. Susskind, “The World as a Hologram”, hep-th/9409089, J. Math. Phys. **36** (1995) 6377.
- [24] L. Susskind and E. Witten, “The Holographic Bound in Anti-de Sitter Space”, hep-th/9805114.
- [25] J. Polchinski, L. Susskind and N. Toumbas, “Negative Energy, Superluminosity and Holography”, hep-th/9903228.